

DISPUTE SETTLEMENT WITH SECOND-ORDER UNCERTAINTY*

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The literature on pretrial dispute settlement has focused on the effect of first-order uncertainty on bargaining, while assuming common knowledge about higher-order beliefs. We study the effect of higher-order uncertainty and show that the existence of such uncertainty improves the efficiency of settlement bargaining by expanding the set of strategies implementable in the equilibrium. We introduce uncertainty about higher-order beliefs by assuming that one player privately observes an imperfect signal of the other player's type. We show that such signals could improve the efficiency only if they are privately observed: The signal's informational value disappears if it is publicly observable.

1. INTRODUCTION

Pretrial dispute settlement has been often studied in the economics literature as a bargaining game under asymmetric information. The information asymmetry considered in the literature is limited to the uncertainty about the first-order beliefs, whereas higher-order beliefs are assumed to be common knowledge. In particular, it is commonly assumed that each disputing party possesses private information about her type while the distribution of types is commonly known to both parties.¹

Asymmetric information about first-order beliefs is a well-known source of inefficiency in bargaining games. One, therefore, might be led to believe that introducing informational asymmetry about higher-order beliefs would further exacerbate bargaining inefficiencies. The central message of this article is to the contrary: Uncertainty about higher-order beliefs could improve the efficiency of a bargaining game.

To make the above point as succinctly as possible, we study a simple signaling game of settlement bargaining under which the type of the defendant determines the likelihood of each party's success in arbitration. We refer to a defendant who is more (less) likely to win in a

*Manuscript received December 2017; revised February 2021.

For their useful comments, we would like to thank Treb Allen, Pol Antras, Kyle Bagwell, Andrew Bernard, Eric Bond, Andrew Daughety, Taiji Furusawa, Jota Ishikawa, Gita Gopinath, Jihong Lee, Marc Melitz, Hiroshi Mukunoki, Jennifer Reinganum, Joel Rodrigue, Kenneth Rogoff, Kathryn Spier, Christopher Snyder, Robert Staiger, Alexander Wolitzky, Laixun Zhao, and the seminar participants at Dartmouth College, Harvard, Kobe, Stanford, Vanderbilt, the Asia Pacific Trade Seminar, and the Midwest Theory Conference. For their detailed and constructive comments, we are grateful to the editor and anonymous referees. We also thank Jun-Tae Park and Sang Min Lee for their research assistance. Beshkar gratefully acknowledges financial support from Fundação para a Ciência e Tecnologia, Portugal under project # PTDC/EGE-IND/31081/2017. Park would like to thank the people at the economics department of Harvard for their hospitality during his visit (2016-2017) as a Fulbright scholar, conducting this research. Park acknowledges the financial support from National Research Foundation of Korea (NRF-2014-S1A3A2043505), Seoul National University Asia Center (Asia Research Foundation Grant #SNUAC-2017-008), and "Overseas Training Expenses for Humanities & Social Sciences" through Seoul National University. Please address correspondence to: Jee-Hyeong Park, Department of Economics, Seoul National University, 1, Gwanank-ro, Gwanak-gu, Seoul 08826, Korea. Email: j-hpark@snu.ac.kr.

¹ There is a growing literature, including Bergemann and Morris (2009), Chen et al. (2017), Morris et al. (2016), that analyzes the effect of higher-order beliefs and associated uncertainty on games and mechanism design problems. The literature on prearbitration settlement, however, has not explored such an issue. For a comprehensive review of the literature on litigation and prearbitration settlement, see Daughety and Reinganum (2017) and Spier (2007).

potential arbitration as the high (low) type. Because the outcome of arbitration is uncertain, risk-averse parties engage in settlement bargaining to avoid arbitration. The threat of arbitration could deter the low-type defendant—who has a relatively higher chance of losing in arbitration—from claiming a high type.

Settlement bargaining is modeled as a signaling game in which the informed party—that is, the defendant—makes a settlement proposal and the uninformed party decides whether to accept it or invoke arbitration. The point of departure of our study is the assumption that although the defendant's type is her private information, the complainant receives a noisy signal about it privately. The private signal observed by the complainant affects his prior belief about the state of the world.²

Therefore, due to the private nature of this signal, the defendant does not precisely know the complainant's belief about the true state of the world. As a result, this bargaining game features second-order uncertainty, namely, the parties have asymmetric information about higher-order beliefs.

Our main finding is related to the role of uncertainty about second-order beliefs on the likelihood and efficiency of settlement. In particular, we find that an imperfect signal of the state of the world increases the rate of settlement as well as the joint welfare of the parties if and only if it is observed privately by the uninformed party. That is, in this class of bargaining games, an informative signal of the state of the world entails no consequences if it is observed by both parties to bargaining.

To obtain a general intuition for this result, note that the complainant's strategy is to use the threat of arbitration to deter the low-type defendant from mimicking a high type. This strategy sometimes results in the undesirable outcome of arbitrating the high-type defendant. A private signal enables the complainant to employ a richer strategy by conditioning its settlement decision on his privately observed signal. Given the informativeness of this signal about the true type of defendant, this richer strategy generates the necessary deterrence for the low-type defendant at a lower cost for the high-type defendant. In particular, given the private signal, the complaining party could reject a high settlement proposal with a higher probability if he receives a low signal. If the signal is informative, this strategy will produce a higher rate of rejection against an imposter's proposal than the one for a genuinely high-type defendant's.

With a public signal, however, it is impossible to have an equilibrium strategy that rejects the proposals of an imposter and a genuinely high-type defendant with different probabilities. Regardless of the realized value of a public signal, (i) the high-type defendant always makes the highest possible settlement offer acceptable to the complainant, and (ii) the complainant rejects such a proposal with the lowest probability that is needed to discourage the low-type defendant from offering the same proposal in a unique separating equilibrium of this settlement bargaining game.³

The original motivating example for our analysis was the Dispute Settlement Process (DSP) of the World Trade Organization (WTO).⁴ The obligations of an importing country under the WTO are contingent on its domestic political economy conditions, which are likely to be the importing government's private information. Other governments, however, could also conduct their own investigations and receive informative signals about the importing country's political economy conditions.⁵ These signals, which are potentially the private information of

² We use "defendant's type" and "state of the world" interchangeably.

³ As in other signaling games, there exist multiple Perfect Bayesian Equilibria. By applying the Universal Divinity refinement of Banks and Sobel (1987), typically employed in the settlement bargaining literature, we show that there exists a unique separating equilibrium.

⁴ To analyze dispute settlement under the WTO, in a previous version of the article we used a model in which each player has an action, corresponding to tariffs imposed by each country. In the current version of the article, we simplify the analysis by assuming that only one of the parties has an action variable. Although using two action variables in the model makes the analysis more complicated, the central message of the article remains unchanged: A private signal of the defendant type enhances the efficiency of the consultation stage of the DSP.

⁵ For example, exporting firms may have some cost information that is common among firms in the same industry, which in turn can be informative in assessing the level of damages inflicted on import-competing firms in their export

the investigating governments, creates second-order uncertainty in the pre-arbitration dispute settlement game.

The WTO Dispute Settlement Understanding, which establishes the rules of dispute settlement in the WTO, requires the disputing parties to engage in a mandatory “bilateral consultation” stage before requesting adjudication by a panel of experts. The manner in which parties interact within this first formal stage of the dispute settlement process is not disciplined by the WTO agreement. Nevertheless, if arbitration is invoked, the evidence that each party produces at the arbitration stage becomes public information. A desire to keep the evidence of the case private is often cited as one reason to settle a dispute in the consultation stage. Among various reasons for a preference for secrecy, our analysis suggests that it is beneficial to keep the defendant in doubt about the complainant’s seriousness to pursue the case by withholding the evidence that the complainant has produced privately.

Our analysis is applicable to other pre-conflict bargaining situations such as peace negotiations under the threat of an armed conflict. In this game, a party’s incentive to accept or reject a proposed peace deal depends on its information about the other party’s military strength, akin to the likelihood of winning in arbitration in the terminology of our paper. In this context, a public signal of military strength may be produced using military maneuvers and the public display of a country’s military equipment and readiness. Alternatively, information obtained through military espionage may be most appropriately understood as private signals because the spied-upon country does not exactly know the type of information that the other country has obtained through espionage. Our result suggests that allowing your enemy to spy on you, thereby obtaining a private signal, may lead to a better outcome than conducting military maneuvers for the purpose of showing off your military might.

On the enforcement of international trade agreements, especially in determining whether retaliations against alleged violations must be authorized, Park (2011) provides a pro-public-monitoring result. Using a repeated-game framework with imperfect monitoring of the potential use of concealed trade barriers, Park (2011) demonstrates that publicizing the imperfect private signal of potential deviations may facilitate a higher level of cooperation by relaxing the incentive constraint associated with utilizing imperfect private signals in invoking punishment.⁶

Together with Park (2011), our analysis provide an explanation for why the WTO may take very different stances in the prearbitration stage and in the arbitration stage with regard to the publicity of information utilized in such procedures. Our analysis also predicts that an improvement in the quality of signals received by the complaining government will reduce the probability of invoking arbitration. This theoretical finding is consistent with the evidence provided by Ahn et al. (2014) who find a positive correlation between a proxy for information asymmetry and the rate at which arbitration is invoked in WTO disputes. The fact that the rate of the WTO disputes has decreased over time may also reflect a reduction in information asymmetry between the parties (i.e., improved signals) after years of partnership.⁷

Our article is closely related to the literature on dispute settlement in the WTO. In particular, Beshkar (2010, 2016), Park (2011), and Maggi and Staiger (2017, 2018) study the role of the WTO as a public signaling device that reveals some useful—albeit imperfect—information

destination. Such cost information is often confidential business information, as illustrated by a large number of “Best Information Available” cases in the U.S. antidumping investigations caused by refusing to submit cost-related information despite the risk of paying excessively high antidumping duties. The government of the exporting country that is subject to a contingent protection measure may obtain such cost information of its exporting firms in evaluating the merit of filing a complaint against the protectionist measure.

⁶ Using a repeated game framework with incomplete information of potentially persistent political pressure for protection, Bagwell (2009) analyzes enforcement issues in trade agreements, demonstrating that a government facing a low political pressure may “pool” and apply its tariff at the bound rate, which is inefficiently high for her.

⁷ The number of WTO dispute cases decreased from 335 during its first 10 years (1995-2005) to 165 during the next 10 years (2006-2015). This decrease in the WTO disputes is even more surprising once we consider the steady expansion of the WTO membership from to 123 countries in 1995 to 162 countries in 2015, including major ones, such as China (2001) and Russia (2012).

about the type or action of the defending party.⁸ A question has been hovering over these studies: How would the value of the court as a public signaling device change if we consider the ability of the uninformed parties to conduct their own investigations to obtain an independent signal about the state of the world or the other party's actions? This question is particularly interesting given that the disputing parties are most likely better equipped than the WTO arbitrators to monitor and extract information about the private type or actions of each other. Our study advances this line of research by shedding light on the impact of private monitoring conducted by the disputing parties.

Our anti-public-signal result may be compared to that of Morris and Shin (2002) who show that an increase in the precision of public information may generate a detrimental effect on the overall welfare of participants in a coordination game when each participant has access to private information. The public information in Morris and Shin (2002) serves as a coordination device among participants, creating the possibility of inducing a weight on the public information that is higher than the socially optimal level. In our signaling game of settlement bargaining, the public information eliminates second-order uncertainty that enables the receiver to make its rejection threat contingent on the information about the fundamentals (i.e., the sender's type), which in turn completely eliminates its informational value.

Our study is distinct from the literature on two-sided private information in which the parties' types are independent. Schweizer (1989) assumes that each disputing party receives an independent signal about the probability of its success in the court. Daughety and Reinganum (1994) also propose a dispute settlement model with two-sided imperfect information under which the complainant is privately informed about the extent of damages incurred, and the defendant is privately informed about the likelihood of being found liable for damages in the court. In both of these studies, the distribution of types is common knowledge and, thus, there is no second-order uncertainty.

Our result that a public signal has no impact on the equilibrium is related to the analysis of Bagwell (1995). He shows that any level of noise in a follower's observation of a first mover's action can induce the follower to completely ignore its imperfect information in a pure-strategy equilibrium, which in turn eliminates the first mover advantage.⁹ Public information without any noise will induce the players to utilize such information in our settlement bargaining game, eliminating the need for invoking arbitration in the equilibrium. In contrast to the game analyzed by Bagwell (1995), the imperfect information of the follower (i.e., the recipient of a settlement offer) retains its informational value as long as it generates second-order uncertainty to the first mover (i.e., the maker of a settlement offer).

The result that common knowledge can reduce the efficiency of bargaining or hinder negotiation arises in other settings as well. Ayres and Nalebuff (1996) provide a series of examples showing how mediators can facilitate an agreement by preventing the creation of common knowledge. They argue that preserving ignorance about higher-order information—which may be achieved by employing a mediator who transmits only first-order information—can promote trade between a buyer and a seller.

Applying mechanism design to international conflict resolution, Hörner et al. (2015) demonstrate how a mediator without enforcement power can replicate the welfare outcome of an optimal settlement mechanism that utilizes an arbitrator with enforcement power. Under their model, the mediator overcomes its lack of enforcement power by choosing a recommendation strategy that does not reveal the type of a weak player to a strong player. One could interpret the optimality of uncertainty in the mediator's recommendation as optimality of second-order uncertainty as we find in this article.

⁸ Another related paper is Maggi and Staiger (2011) in which the arbitrator is modeled as an arbitrator who interprets ambiguous obligations, fills gaps in the agreement, and modifies rigid obligations. See Park (2016) for a comprehensive review of the recent literature on trade disputes and settlement.

⁹ Maggi (1999) demonstrates that the strategic value of commitment (e.g., moving first) is restored even with imperfect observability of commitment when a leader has private information about her type.

In Section 2, we describe the basic setup of our pretrial bargaining model. In Section 3, we characterize the equilibrium of this game under public and private signals, respectively, and analyze the role of second-order uncertainty by comparing these two equilibria. We discuss the robustness of our main result in Section 4. Finally, we provide some concluding remarks in Section 5.

2. THE BASIC SETUP

In this section, we introduce the basic setup of our pre-arbitration bargaining model. There are two parties: (*D*)efendant and (*C*)omplainant. *D* has an action variable $t \in T \subset R^+$. Payoffs of *D* and *C* are denoted by $u(t; \theta)$ and $v(t)$, respectively, where θ represents the state of the world, or *D*'s type. We consider payoff functions under which *D*'s unilaterally-optimal action exceeds the one that maximizes the joint payoffs of the parties. Namely, letting $t^N(\theta) \equiv \arg \max_{t \in T} u(t; \theta)$ and $t^E(\theta) \equiv \arg \max_{t \in T} (u(t; \theta) + v(t))$ denote, respectively, the noncooperative and jointly-efficient levels of t , we assume that

$$t^N(\theta) > t^E(\theta).$$

For a continuous action set, T , this condition is satisfied if:

1. *D*'s payoff is concave and initially increasing in t :

$$\frac{\partial^2 u}{\partial t^2} < 0, \quad \left. \frac{\partial u}{\partial t} \right|_{t=0} > 0.$$

2. *D*'s action has a negative externality on the other party:

$$\frac{\partial v}{\partial t} < 0.$$

3. The marginal payoff of *D* from raising her own action is increasing in the state parameter, θ , which can take one of two levels, high (h) and low (l), with $h > l$.¹⁰

$$\frac{\partial u(t; h)}{\partial t} > \frac{\partial u(t; l)}{\partial t}.$$

4. The joint payoff is concave in t :

$$\frac{\partial^2 (u + v)}{\partial t^2} < 0.$$

We further assume that a high state, h , realizes with probability ρ (thus, l with probability $1 - \rho$). Therefore,

$$\begin{aligned} t^N(h) &> t^N(l), \\ t^E(h) &> t^E(l). \end{aligned}$$

Finally, we assume that

$$t^N(l) > t^E(h).$$

¹⁰ This assumption implies that *D* has a greater marginal utility from a higher action under the high state of the world, $\theta = h$. In the context of trade disputes, for example, $\theta = h$ represents a high political pressure for protection that a government faces, which increases its political payoff from choosing a higher level of import protection.

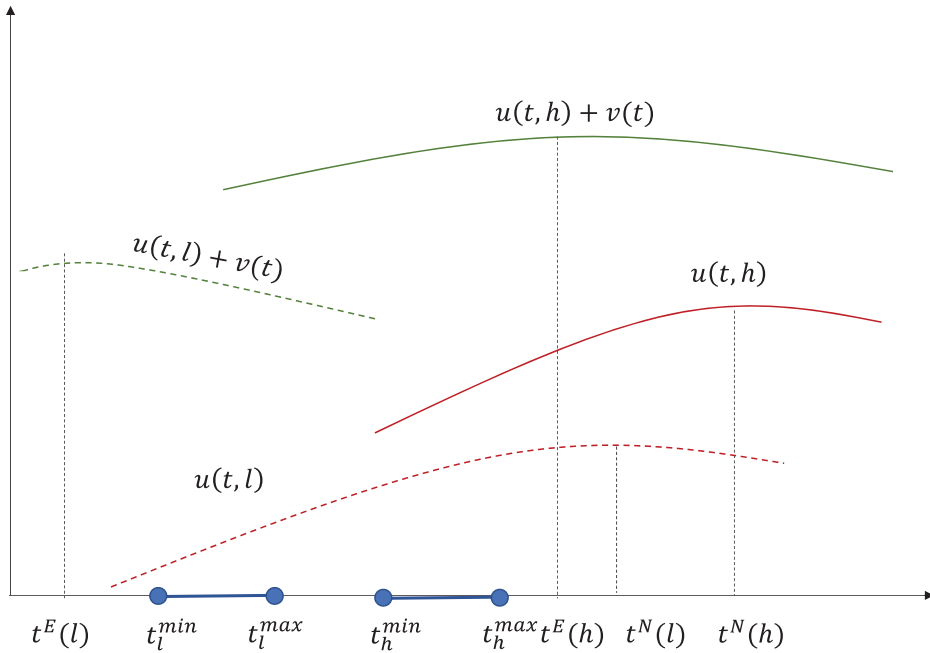


FIGURE 1
PAYOFFS

This last assumption is made to avoid a taxonomy of cases by eliminating the possibility of commitment overhang under an optimal agreement.¹¹ Figure 1 shows the defendant’s payoffs and the joint payoffs as functions of t and θ . $t^N(\theta)$ and $t^E(\theta)$ are also depicted in this figure.

2.1. *Information Structure.* While D observes θ privately, C receives an imperfect signal of θ , denoted by θ^C , which is accurate with probability γ , namely,

$$\Pr(\theta^C = l|\theta = l) = \Pr(\theta^C = h|\theta = h) = \gamma \in \left(\frac{1}{2}, 1\right).$$

The arbitrator, A , also receives a signal, denoted by θ^A , with accuracy γ^A , namely,

$$\Pr(\theta^A = h|\theta = h) = \Pr(\theta^A = l|\theta = l) = \gamma^A \in \left(\frac{1}{2}, 1\right).$$

We do not make any assumption regarding the relative accuracy of the signals observed by the complainant and the arbitrator, that is, γ and γ^A . We, however, assume that these signals are independent conditional on θ .

In Subsection 3.1, we assume that the signal θ^C is publicly observed by both C and D . In Subsection 3.2, we assume that this signal is privately observed by C .

¹¹ Commitment overhang occurs when a party is allowed to take an action that is lower than the level designated in the contract. In such cases, this party would not utilize the maximum action allowable, thereby generating a commitment overhang. In our current context, suppose that $t^N(l) < t^E(h)$ and that D has committed not to choose an action above $t^E(h)$. It follows that in this case a low-type D will choose its ideal action, $t^N(l)$, thereby generating a commitment overhang equal to $t^E(h) - t^N(l)$. For an analysis of commitment overhang, see Amador and Bagwell (2013); Beshkar et al. (2015); Beshkar and Bond (2017).

The important difference between private and public signals is that under a private signal (and only under a private signal), there is second-order uncertainty in the relationship between the disputing parties: If the signal is private, the defendant faces uncertainty about the complainant's belief about the defendant's type.

2.2. Arbitration. We model the arbitrator as a court that, if invoked, issues a binding ruling as a function of its informative signal, θ^A . The arbitrator's ruling is an action $t_A(\theta^A)$ to be implemented by D . Assuming that the objective of the arbitrator is to maximize the expected joint payoff of the parties given θ^A , we have

$$(1) \quad t_A(\theta^A) \equiv \arg \max_t \sum_{\theta=l,h} \Pr(\theta|\theta^A)[u(t; \theta) + v(t)].$$

By design, the arbitrator is non-Bayesian, meaning that she will not use any information that may be inferred from the actions of the players in the pre-arbitration stage. This assumption helps us focus on the issue of second-order uncertainty, and it may also be justified by legal practices as well as previous results in the literature that point to the benefits of avoiding the use of information from settlement bargaining as evidence in the court. For example, according to Rule 408 of the U.S. Federal Rules of Evidence, proposed compromises by the parties in an attempted dispute resolution negotiation are generally inadmissible as evidence in the court. Using a signaling model with one sender (the defendant) and two receivers (the plaintiff and the court), Daughety and Reinganum (1995) analyze the consequences of admitting pretrial settlement proposals as evidence in the court.¹² They find support for the proposition that inadmissibility of pretrial negotiations as evidence in the court may serve public interests, although it may not benefit all parties.¹³

The arbitration rule (1) has a number of intuitive implications. First, it implies that the arbitrator assigns a higher action when a higher state of the world is observed, that is, $t_A(h) > t_A(l)$, which ensures that a genuinely high-type defendant is more likely than an imposter to receive a favorable ruling in arbitration.

Second, under any realization of θ , there are actions that both parties prefer to arbitration. To state this formally, let $u_A(\theta)$ and $v_A(\theta)$ denote, respectively, the expected payoffs of D and C from arbitration when the true state of the world is θ . Moreover, let t_θ^{\min} (t_θ^{\max}) denote the action that makes $D_\theta(C)$ indifferent about settlement and arbitration when the true state of the world is θ , that is,

$$(2) \quad \begin{aligned} u(t_\theta^{\min}; \theta) &\equiv u_A(\theta), \\ v(t_\theta^{\max}) &\equiv v_A(\theta). \end{aligned}$$

We can then show that

LEMMA 1. $t_\theta^{\max} > t_\theta^{\min}$.

This lemma results from the uncertainty in the arbitration outcome together with the concavity of the joint payoff. This lemma implies that if the state of the world is publicly known to be θ , both parties prefer settlement with any $t \in [t_\theta^{\min}, t_\theta^{\max}]$ to arbitration. These ranges of actions for possible settlement are demonstrated in Figure 1.

¹² The analysis of a Bayesian arbitrator is considerably more complicated in our model. That is because in addition to having two receivers (i.e., the defendant and the arbitrator), in our framework there are also two senders (i.e., the complainant and the defendant.)

¹³ Also see Daughety and Reinganum (2014) for the review of the literature on the revelation and suppression of private information in settlement-bargaining models.

Third, our assumption about the arbitration rule implies that arbitration is useful, namely, it generates a higher expected joint welfare than playing a fixed action under all states of the world, namely,

$$(3) \quad E_\theta[u_A(\theta) + v_A(\theta)] > E_\theta[u(t; \theta) + v(t)]$$

for all $t \in T$. If, on the contrary, there exists t that violates the above inequality, the arbitrator could increase the expected ex ante joint welfare of the parties by ruling t instead of $t_A(\theta^A)$, which is a contradiction to the definition of $t_A(\theta^A)$ in (1) that entails $t_A(l) < t_A(h)$.

Note that the optimality of the arbitration process is only a sufficient—but not necessary—condition for the inequality (3) to hold. In particular, the subsequent analysis will hold under any arbitration rule that satisfies this condition.

Finally, we make the intuitive assumption that given a settlement proposal, t , the loss from arbitration for the low-type defendant is larger than the loss for the high-type defendant, that is, $u(t, l) - u_A(l) > u(t, h) - u_A(h)$ for all t . Together with our earlier assumption that $\frac{\partial u(t;h)}{\partial t} > \frac{\partial u(t;l)}{\partial t}$, this assumption implies

$$(4) \quad \frac{u(t; l) - u_A(l)}{u(t; h) - u_A(h)} > \frac{\frac{\partial u(t;l)}{\partial t}}{\frac{\partial u(t;h)}{\partial t}}.$$

2.3. The Signaling Game of Settlement Bargaining. To analyze the settlement bargaining problem, we use a signaling model in which D 's settlement proposal signals its type. The key difference with a standard signaling model is that the uninformed party (C) also receives a signal of the state of the world, that is, the informed party's type. The sequence of events is as follows:

1. State of the world, θ , is realized and observed privately by D .
2. C receives an imperfect signal, θ^C , of the state of the world. In Subsection 3.1, we assume that this signal is observed by both C and D . In Subsection 3.2, we assume that this signal is privately observed by C .
3. D proposes an action, $t \in T$, for settlement.
4. C either accepts t (and t is implemented), or rejects t and the dispute escalates to arbitration.
5. If arbitration is invoked, the arbitrator receives a signal, θ^A , and enforces an action, $t_A(\theta^A)$.

If θ^C is observed privately by C , the defendant does not accurately know the complainant's belief about the defendant's type, thereby generating a second-order uncertainty in the relationship between the disputing parties. In other words, under a private signal, C 's belief about D 's type is not common knowledge. Conversely, if both parties observe θ^C , there will be no second-order uncertainty because C 's information about D 's type will be common knowledge.

To understand how private signals may affect settlement negotiation, we ask whether the efficiency of settlement bargaining improves if the second-order uncertainty were eliminated, that is if the private signal were publicized. In Subsection 3.3, we present Theorem 1, which establishes that the answer to this question is negative, namely, an informative signal about D 's private information is useful if and only if it is observed privately by C .

A full analysis of the value of second-order uncertainty is deferred until Subsection 3.3, but at this point we can offer a general intuition about this result: Suppose that the complaining party chooses to reject a settlement proposal with a higher probability if he receives a low signal. If the signal is informative, this strategy will produce a higher rate of rejection against an imposter than the one for a genuinely high-type defendant. Under a public signal, however, it is impossible to have an equilibrium strategy that rejects the proposals of an imposter and a genuinely high-type defendant with different probabilities. Regardless of the realized value of

a public signal, (i) the high-type defendant always makes the highest possible settlement offer acceptable to the complainant, and (ii) the complainant rejects such an offer with the lowest probability that is needed to discourage the low-type defendant from mimicking the high type.

3. EQUILIBRIUM

To find the equilibrium, we use the notion of Perfect Bayesian Equilibrium (PBE). To construct a PBE, note that regardless of his observed signal, C will accept any proposal lower than or equal to t_l^{\max} and rejects any proposal higher than t_h^{\max} , as defined in (2). Therefore, D_h will only propose actions in the $[t_h^{\min}, t_h^{\max}]$ range and D_l will only propose either t_l^{\max} or actions in $[t_h^{\min}, t_h^{\max}]$.

As is common in signaling games, this game has multiple PBEs. To weed out implausible PBEs, we use the Universal Divinity refinement of Banks and Sobel (1987). In a nutshell, this refinement dismisses a PBE as unreasonable if a high-type D is more likely than a low-type D to gain from deviation to an off-equilibrium action. That is because in that case, a high-type D could differentiate herself from a low-type D by deviating from the equilibrium, thereby inducing a more favorable response from the receiver (C). This refinement produces a unique equilibrium under both versions of the settlement bargaining game with public and private signals.

This game, under both public and private signals, may have pooling PBEs. However, we show in the Appendix—and it is also well-known in the literature on signaling games (Reinganum and Wilde, 1986; Cave, 1987)—that pooling PBEs do not pass the divinity refinement.¹⁴ In what follows, therefore, we focus on separating equilibria.

3.1. Equilibrium under a Public Signal. Before introducing our analysis of the settlement bargaining game under a private signal and second-order uncertainty, it is instructional to characterize the equilibrium under a public signal in which there is no second- (or higher-) order uncertainty. A public signal simply updates the common prior of the parties about the state of the world. In particular, letting $\rho^*(\theta^C)$ denote the updated common prior of the parties given θ^C , we have

$$\rho^*(\theta^C) \equiv \Pr(\theta = h | \theta^C) = \rho \frac{\Pr(\theta^C | \theta = h)}{\Pr(\theta^C)}.$$

Beyond determining the common prior of the parties, θ^C has no effect on the bargaining game.¹⁵ The strategy profile is given by

$$(5) \quad (\alpha_l(t), \alpha_h(t), \beta(t)),$$

where $\alpha_\theta(t)$, $\theta \in \{l, h\}$, is the probability that D_θ assigns to proposing t for settlement, and $\beta(t)$ is the probability with which C rejects the proposed settlement, t .¹⁶ Taking D 's strategy

¹⁴ See Lemma A.1 for a proof.

¹⁵ This public signal affects the publicly available information, that is, the common belief about the state of the world. The public signal potentially affects the strategy of each player by affecting the complainant's belief as if the common prior of the parties has been updated.

¹⁶ More precisely, the rejection probability should be denoted by $\beta_{\theta^C}(t)$ because the rejection probability could in general depend on θ^C . However, we show that in equilibrium $\beta_l(t) = \beta_h(t)$ and, thus, we drop the subscript for brevity.

as given, C forms a belief about the type of D as a function of t . For $t \in \{t_l^{\max}\} \cup [t_h^{\min}, t_h^{\max}]$, this belief follows the Bayes' law, namely,¹⁷

$$(6) \quad \Pr(\theta = l|t) = 1 - \Pr(\theta = h|t) = \frac{\alpha_l(t)(1 - \rho^*)}{\alpha_l(t)(1 - \rho^*) + \alpha_h(t)\rho^*}.$$

Therefore, given D 's equilibrium strategy, C will accept a proposed settlement, t , only if

$$(7) \quad v(t) \geq \Pr(\theta = l|t)v_A(l) + \Pr(\theta = h|t)v_A(h),$$

where the right-hand side (RHS) represents C 's expected payoffs from arbitration.

In a separating equilibrium, D_l must (weakly) prefer to propose t_l^{\max} than t . This condition can be written as

$$(8) \quad u(t_l^{\max}; l) \geq \beta(t)u_A(l) + [1 - \beta(t)]u(t; l) \quad \forall t.$$

Conditions (7)–(8) together with the Universal Divinity refinement pin down the equilibrium of the game as described in the following proposition:

PROPOSITION 1. *In the settlement bargaining game without higher-order uncertainty, there is a unique equilibrium in which the two types of defendant separate: D_l proposes t_l^{\max} and D_h proposes t_h^{\max} . C accepts the former and rejects the latter with a positive probability, $\beta(t_h^{\max}) > 0$, that leaves D_l indifferent between proposing t_l^{\max} and t_h^{\max} —that is, $\beta(t_h^{\max})$ is uniquely determined by (8) holding with equality.*

Notice that the common prior of the parties, ρ^* , which is determined by the public signal, has no impact on the equilibrium.¹⁸ Therefore, as long as the public signal is imperfect, the equilibrium is independent of the public signal or its accuracy.

This result comes directly from the following two characteristics of the equilibrium strategy: Regardless of the realized value of a public signal, (i) a high-type defendant always makes the highest possible settlement offer that is acceptable to the complainant, t_h^{\max} ,¹⁹ (ii) in order to discourage a low-type defendant from offering the same proposal, C must always reject such a proposal with a certain probability that is determined by (8).

A separating equilibrium implies that first-order uncertainty is resolved in the equilibrium of this game. However, it is the lack of second-order uncertainty that renders the signal received by the uninformed party irrelevant. In particular, as shown in the next subsection, although first-order uncertainty is once again resolved in a separating equilibrium, the uninformed party's signal will affect the equilibrium if there is second-order uncertainty.

3.2. Equilibrium under a Private Signal. In contrast to a public signal, which updates the common prior of the parties about the defendant's type, a private signal does not change the common prior: It only updates the complaining party's prior/belief. Therefore, a private signal creates second-order uncertainty: The defendant is uncertain about the signal received by the complainant and, hence, about the complainant's belief about the defendant's type.

¹⁷ Recall that proposing $t \notin \{t_l^{\max}\} \cup [t_h^{\min}, t_h^{\max}]$ cannot be a PBE.

¹⁸ The assumption that parties have a common prior is for simplicity. The results in this article will continue to hold if we assume heterogeneous but commonly-known priors. We thank a referee for pointing this out.

¹⁹ Assumption (4) together with the incentive compatibility condition (8) imply that the high-type defendant's expected payoff is increasing in the settlement proposal, t . Thus, offering t_h^{\max} maximizes the high-type defendant's expected payoff among all separating PBEs. Offering $t \in [t_h^{\min}, t_h^{\max})$ does not satisfy the Universal Divinity refinement. That is because if the defendant deviates from the equilibrium strategy by offering a $t' > t$, C would believe that such an off-the-equilibrium action is taken by a high-type defendant, which in turn makes C to accept such a deviatory offer for sure. See the proof of Proposition 1 in the Appendix for a more detailed discussion.

The strategy profile under a private signal is given by $(\alpha_l(t), \alpha_h(t), \beta_l(t), \beta_h(t))$, where $\alpha_\theta(t)$, $\theta \in \{l, h\}$, is the probability that D_θ assigns to proposing t for settlement, and $\beta_{\theta^C}(t)$ is the probability with which C_{θ^C} will reject t as a settlement proposal.

Taking D 's strategy, $\alpha_\theta(t)$, as given, C forms a belief about D 's type as a function of the proposed settlement and his privately observed signal. This belief can be presented as a conditional probability function, $\Pr(\theta|\theta^C, t)$, which is the likelihood that D 's type is θ conditional on t and θ^C . C_{θ^C} will accept a settlement proposal, t , only if

$$(9) \quad v(t) \geq \Pr(\theta = l|\theta^C, t)v_A(l) + \Pr(\theta = h|\theta^C, t)v_A(h),$$

where the RHS represents C_{θ^C} 's expected payoff from arbitration conditional on θ^C and the proposed settlement, t .

In making her settlement proposal, D_θ maximizes the weighted sum of her expected payoff from arbitration, $u_A(\theta)$, and her payoff under a potential settlement, where the weights are the probability of arbitration and settlement induced by the proposed settlement. From the perspective of D_θ , the likelihood that her settlement proposal, t , will be rejected by C is given by $\gamma\beta_\theta(t) + (1 - \gamma)\beta_{\theta^*}(t)$ with $\theta^* \neq \theta \in \{l, h\}$, where γ is the accuracy of the signal observed by C . Therefore, the expected payoff of D_θ from proposing t can be written as

$$(\gamma\beta_\theta(t) + (1 - \gamma)\beta_{\theta^*}(t))u_A(\theta) + [1 - (\gamma\beta_\theta(t) + (1 - \gamma)\beta_{\theta^*}(t))]u(t; \theta).$$

A separating equilibrium will occur only if D_l is weakly better off by separating herself from D_h , namely, if

$$(10) \quad \begin{aligned} u(t_l^{\max}; l) &\geq [\gamma\beta_l(t) + (1 - \gamma)\beta_h(t)]u_A(l) \\ &+ [1 - \gamma\beta_l(t) - (1 - \gamma)\beta_h(t)]u(t; l) \forall t. \end{aligned}$$

The RHS of this condition is the expected payoff to D_l from proposing t .

Equilibrium conditions (9)–(10), together with the Universal Divinity refinement, pin down the equilibrium of the settlement bargaining game under a private signal as follows:

PROPOSITION 2. *Under the equilibrium of the settlement bargaining with an imperfect private signal,*

- (i) D_l and D_h propose t_l^{\max} and t_h^{\max} , respectively,
- (ii) C_{θ^C} always accepts t_l^{\max} but rejects t_h^{\max} with probability $\beta_{\theta^C} \equiv \beta_{\theta^C}(t_h^{\max})$ for $\theta^C = \{l, h\}$, where $\beta_{\theta^C}(t_h^{\max})$ satisfies (10) with equality when $t = t_h^{\max}$ and $(1 - \beta_l)\beta_h = 0$.
- (iii) $\exists \bar{\gamma} \in (\frac{1}{2}, 1)$ such that if $\gamma < \bar{\gamma}$ then $\beta_l = 1$, $\beta_h > 0$, and $\frac{d\beta_h}{d\gamma} < 0$, and if $\gamma > \bar{\gamma}$ then $\beta_l < 1$, $\beta_h = 0$, and $\frac{d\beta_l}{d\gamma} < 0$.

The last part of this proposition describes the way in which the accuracy of the private signal affects the equilibrium. In particular, it implies that

COROLLARY 1. *An increase in the private signal's accuracy, γ , will (i) increase the equilibrium probability of accepting the proposal of a genuinely high-type defendant; (ii) have no impact on the probability of accepting an imposter's proposal or on C 's expected payoff; (iii) increase the expected joint payoff of the parties.*

Note that there is no first-order uncertainty (about the defendant's type) in equilibrium, which is true for the defendant by construction and for the complainant due to the separating nature of the defendant's strategy. Moreover, in equilibrium only the high-type defendant is affected by second-order uncertainty because the unobservable belief of C about the state

of the world determines the likelihood of settlement only when a high offer is made. As a result, the high-type defendant alone reaps all the benefit of the reduction in second-order uncertainty, which comes from an improvement in the accuracy of C 's signal.²⁰

A reduction in second-order uncertainty raises the high-type defendant's expected payoff through the following economic mechanism: As shown in part (iii) of Proposition 2, the complainant receiving a low signal rejects the high-type defendant proposal with a higher rate than the one receiving a high signal ($\beta_l > \beta_h$). With a more accurate signal of D 's type (implying a lower second-order uncertainty), thus the high-type defendant has his proposal rejected with smaller probability.²¹

3.3. *The Value of Second-Order Uncertainty.* Having characterized the equilibria under public and private signals in Propositions 1 and 2, we are now in a position to establish our main result:

THEOREM 1. *An informative signal about D 's private information improves the expected payoffs of the parties in the signaling game if and only if it is privately observed by C .*

The necessity of a private signal (generating second-order uncertainty) for welfare improvement is proven by Proposition 1, which establishes that under a public signal (implying no second-order uncertainty) the equilibrium is unaffected by an imperfect public signal. Moreover, Corollary 1 establishes the sufficiency claim in Theorem 1 by showing that an increase in the accuracy of the private signal—which is associated with a reduction in second-order uncertainty—increases the expected payoffs of the high-type D as well as the expected joint payoffs of the parties.²²

The following thought experiment provides some perspective on the result that a signal is useful only if it is private. Consider an action $t \in [t_h^{\min}, t_h^{\max}]$, and the equilibrium strategies that support them as a separating PBE under public and private signals, respectively. Letting β denote the likelihood of arbitration if t is proposed under a public signal, the incentive compatibility constraint for D_l is given by

$$u(t_l^{\max}; l) \geq \beta u_A(l) + (1 - \beta)u(t; l).$$

Solving for β that makes D_l indifferent yields

$$\beta = \frac{u(t; l) - u(t_l^{\max}; l)}{u(t; l) - u_A(l)}.$$

The value of β is the likelihood that D_h , as well as untruthful D_l , will face arbitration in the equilibrium under a public signal.

In the case of a private signal, recall that the probability of arbitration for an untruthful D_l is given by $\gamma\beta_l + (1 - \gamma)\beta_h$ and, thus, the incentive compatibility constraint is given by

$$u(t_l^{\max}; l) \geq (\gamma\beta_l + (1 - \gamma)\beta_h)u_A(l) + (1 - (\gamma\beta_l + (1 - \gamma)\beta_h))u(t; l).$$

Solving for $\gamma\beta_l + (1 - \gamma)\beta_h$ yields

$$\gamma\beta_l + (1 - \gamma)\beta_h = \frac{u(t; l) - u(t_l^{\max}; l)}{u(t; l) - u_A(l)} = \beta.$$

²⁰ A referee has provided this intuitive explanation for this result.

²¹ C 's rejection rates also fall with a more accurate signal of D 's type, as shown in (iii) of Proposition 2 with $d\beta_h/d\gamma < 0$ and $d\beta_l/d\gamma < 0$.

²² While first-order uncertainty of D 's type is resolved in the separating equilibrium, this fundamental uncertainty still forces C to reject presumably the high-type D 's proposal with a positive probability in equilibrium regardless of C 's signal being private or public.

Therefore, the incentive compatibility constraint requires the same likelihood of arbitration for the low type D under private and public signals.

Now consider the likelihood of arbitration for D_h under a private signal, which is given by $\gamma\beta_h + (1 - \gamma)\beta_l$. For $\gamma > \frac{1}{2}$, we have

$$\gamma\beta_h + (1 - \gamma)\beta_l < \gamma\beta_l + (1 - \gamma)\beta_h = \beta,$$

which implies that D_h faces a lower likelihood of arbitration under a private signal than under a public signal.²³

This comparison clarifies the source of efficiency improvement under a private signal: When the signal is unobservable to D , C can condition its arbitration strategy on the private signal and take different types to arbitration with different probabilities. In particular, under a private signal, D_h is less likely to end up in arbitration than D_l that pretends to be D_h . In contrast, under a public signal an imposter and a genuinely high-type D are equally likely to be taken to arbitration.

4. FURTHER DISCUSSION

In this section, we discuss the robustness of the main result in Section 3 as well as its extensions.

4.1. Almost-Perfect Signals. As emphasized in Theorem 1, regardless of how accurate an imperfect signal is, it will have no effect on the bargaining equilibrium if it is observed publicly. The comparison between private and public signals is the most striking when the signal is almost perfect (i.e., $\gamma \rightarrow 1$): As the signal becomes more accurate, the probability that a high-type defendant faces arbitration goes to zero only if the signal is private. In contrast, when the signal is public, the probability of invoking arbitration does not depend on the quality of the (imperfect) signal.

The reason for this surprising result is that a Universally Divine PBE is always a *separating* equilibrium. In particular, note that a pooling PBE under a public signal is more efficient the more accurate is the signal.²⁴ However, as we show in the Appendix, no pooling equilibrium survives the Universal Divinity refinement. Nevertheless, a high-type defendant's gain from deviating from a pooling equilibrium approaches zero as γ tends to 1.²⁵ This observation leads us to question the appropriateness of the Universal Divinity refinement for this limiting case. Our proposition about the inconsequence of a public signal, therefore, should be viewed as applicable to cases in which signals are subject to nonnegligible errors.

4.2. Uninformed Party Proposing a Settlement. So far we have analyzed second-order uncertainty under the assumption that the informed party (with her type affecting the arbitration outcome, i.e., D) makes a settlement proposal that can be accepted or rejected by the uninformed party (i.e., C). If the *uninformed* party was the one to make a settlement proposal, what would be the role of second-order uncertainty? As we now show, if the settlement proposal is made by the uninformed party, second-order uncertainty would have no impact on the equilibrium.

²³ As shown by Lemma A.2, $\beta_h(1 - \beta_l) = 0$ so that $0 < \beta_l(t) < 1$ implies $\beta_h(t) = 0$ and $\beta_h(t) > 0$ implies $\beta_l(t) = 1$, which in turn implies $\beta_l(t) > \beta_h(t)$. The above inequality results from this inequality together with $\gamma > \frac{1}{2}$.

²⁴ In an earlier version of our article, Beshkar and Park (2019), we characterize such a pooling equilibrium.

²⁵ A referee raises this issue. For any $\gamma < 1$ under a public signal, the referee also points out that there is no exact equilibrium but an ε -equilibrium of Monderer and Samet (1989) which restores approximately the efficiency level under a private signal.

To show this result, consider the following sequence of events. First, C receives a signal of D 's type and proposes an action, t , for settlement.²⁶ D can accept this proposal and settle, or invoke arbitration. This enables us to analyze how changing the party who makes a take-it-or-leave-it offer from an informed party to an uninformed one affects the role that second-order uncertainty plays in the pre-arbitration bargaining game.²⁷

D_θ accepts a settlement proposal, t , if $u(t; \theta) \geq u_A(\theta)$ and invokes arbitration otherwise. Thus, D_θ will accept $t \geq t_\theta^{\min}$. Given this behavior of the defendant, the complainant would either offer t_l^{\min} (which would be accepted only by D_l) or t_h^{\min} (which would be accepted by all types of D). Finally, whether t_l^{\min} or t_h^{\min} is offered in the equilibrium depends on the complainant's belief of the relative frequency of low-type and high-type defendants. Formally,

PROPOSITION 3. *Suppose that the uninformed party (C) makes a settlement proposal that can be accepted or rejected by the informed party (D). Then:*

(i) *For $\Pr(\theta = h|\theta^C) > \frac{v(t_l^{\min}) - v(t_h^{\min})}{v(t_l^{\min}) - v_A(h)}$, there exists a unique PBE in which C proposes t_h^{\min} for settlement and both types of D will accept this proposal.*

(ii) *For $\Pr(\theta = h|\theta^C) < \frac{v(t_l^{\min}) - v(t_h^{\min})}{v(t_l^{\min}) - v_A(h)}$, there exists a unique PBE in which C proposes t_l^{\min} for settlement, which will be accepted (rejected) by D_l (D_h).*

Proposition 3 indicates that if the settlement proposal is made by the uninformed party, second-order uncertainty is inconsequential, that is, it does not matter whether C 's signal is private or public. Given a take-it-or-leave-it offer of the complainant, the defendant ends up only caring about whether or not taking the single action (i.e., offer) is beneficial for her. The first-order uncertainty still matters as it affects the probability distribution that C assigns to D 's type. If C 's signal is very noisy, that is, if $\Pr(\theta = h|\theta^C = h) < [v(t_l^{\min}) - v(t_h^{\min})]/[v(t_l^{\min}) - v_A(h)]$, then C would always propose t_l^{\min} regardless of the realization of the signal. If his signal is sufficiently accurate, then C 's strategy would be to propose t_h^{\min} if $\theta^C = h$ and t_l^{\min} otherwise.

We, therefore, have established that the relevance of second-order uncertainty depends critically on whether the settlement proposal is made by the informed or uninformed party. In the case of pre-arbitration settlement bargaining under the WTO, the signaling game setup is a more natural modeling choice: The decision to raise protection beyond the binding tariff of the agreement is made by the government of the importing country (the informed party) who claims to have faced certain contingencies that are specified in the agreement. In the terminology of signaling games, this decision sends a message about the prevailing state of the world in the importing country. As a result, the decision to invoke arbitration is naturally made by the affected exporting countries who are uninformed about the true state of the world in the importing country.²⁸

Our results in this section are related to the previous literature on the screening game. By analyzing how a principal's "ex ante nonverifiable signal" improves contracting, Laffont and Martimort (2002) show that the principal shuts down the inefficient agent if her signal implies that it is sufficiently likely that the agent is inefficient.²⁹ This result corresponds to

²⁶ If the parties had the option to make side payments, the complaining party would have been able to offer a menu of settlement proposals that the defending party could choose from. In the absence of side payments, as we assume in this article, the equilibrium settlement proposal consists of only one action.

²⁷ Thus, this subsection does not intend to analyze how an uninformed party's noisy signal of an informed party's type affects the outcome of a more general screening game with side payments, which is beyond the scope of this article. As a related study in this direction, Severinov (2008) analyzes the mechanism design problem in which an informed principal has a private signal that is correlated with agents' types. He shows that the principal can appropriate all expected social surplus from an ex post efficient solution under two informational requirements that are generic with at least two agents (excluding the principal).

²⁸ We provide a detailed discussion of trade dispute settlements and modeling choices in Section 5 of an earlier version of this article, Beshkar and Park (2019).

²⁹ See Section 2.14.2 on "Ex Ante Nonverifiable Signal" on p. 70 of Laffont and Martimort (2002) for this result.

Proposition 3(ii) that under a sufficiently accurate signal, the complainant makes an offer that is only acceptable to a low-type defendant.

The equilibrium that arises when the signal is too inaccurate (Part (i) of Proposition 3) is different from the result of Laffont and Martimort (2002) in which an efficient agent takes an efficient action with some informational rent whereas an inefficient agent takes an inefficient action with no rent, a typical result under an optimal contract. Our unconventional result hinges on the assumption that the complainant cannot offer a payment in association with an action to be taken by the defendant, which in turn enables the low-type defendant (i.e., an efficient agent) to always profitably take an action that the high type would take.³⁰ Finally, our analysis in this subsection may be compared to the screening model of Maskin and Tirole (1990) with an informed principal and private values.³¹ In their screening game, the informed principal proposes a mechanism to the agent who may accept or reject it. Maskin and Tirole find that the principal is generically better off if the signal she receives is privately observed. This result is in contrast to our finding in Proposition 3 that whether *C*'s signal is private or public is inconsequential if the proposal is made by the uninformed party.

In the screening game of Maskin and Tirole (1990), the agent's incomplete information enables a principal of a given type to raise her payoff above the full-information level by allowing her to violate the individual rationality (IR) and incentive constraint (IC) conditions, with these violations being offset by the other types' IR and IC conditions.³² Note, however, that their analysis does not apply to our setting in which the mechanism, that is, the arbitration process, is predetermined and it is not up to the parties to change it. The proposal of the principal (i.e., the complainant in our setting) is limited to a take-it-or-leave-it settlement offer so that exchanging the slack on the agent's IR and IC conditions among principals of different types are not allowed.

5. CONCLUSION

We consider the role of second-order uncertainty in a signaling game of settlement bargaining. The second-order uncertainty is generated if the uninformed party receives a private signal of the informed party's type. We find that, under some weak conditions, the additional information provided by this signal affects the equilibrium of the signaling game if and only if it is privately observed by the uninformed party—that is, if it generates second-order uncertainty. Moreover, this private signal increases the efficiency of the bargaining outcome by increasing the likelihood that a settlement is achieved and arbitration is avoided.

A potentially interesting direction for future work is to extend our setup to include an alternating-offers bargaining.³³ This is especially important for the study of foot-dragging in a dispute resolution process in which different parties may experience different costs and benefits from delaying the resolution of the dispute. In the case of WTO disputes, for example, the defending country may have an incentive to delay the resolution of the dispute because during this process the defending country could apply its disputed policy without punishment. In an alternative setup in which a proposed action cannot be applied until the resolution of the dispute, the complaining country would benefit from delay in bargaining.

³⁰ Thus, this equilibrium of both types taking the same action arises in our setting unless the complainant's signal implies that shutting down the inefficient agent is beneficial for him.

³¹ In Maskin and Tirole (1990), the principal has private information of her type that, given an action, does not affect the agent's expected payoff.

³² When the agent knows the principal's realized type (the case of "full information"), then the IR and IC conditions must hold individually for each type of principals. If the agent does not know the principal's type, then these constraints only need to hold in expectation over the unknown types. See the intuitive explanation on p. 390 of Maskin and Tirole (1990) (about their Proposition 1) for a more detailed explanation of this result.

³³ The presence of second-order uncertainty makes this extension nontrivial. As shown in Subsection 4.2, changing the order of who makes a take-it-or-leave-it offer affects the role of second-order uncertainty in bargaining, thus an alternating-offers bargaining may also affect it as well.

We assumed that the production of the imperfect signal is automatic and costless. How would our results change if the uninformed party could decide whether or not to produce an imperfect signal, potentially at a cost? This is especially important to consider in future research given that the efficiency gains from the signal accrues to the informed party.³⁴ This observation also suggests that the informed party has the incentive to generate or facilitate the generation of an imperfect private signal—for example, by turning a blind eye to espionage attempts by the adversaries.

Finally, one may also consider introducing a possible predispute information gathering/exchanging stage to our model.³⁵ Such an extension would be useful for analyzing the incentives of the informed party to generate a public signal of her type. Suppose the defendant had the opportunity to provide the hard public evidence of her type, under which conditions would she do it or not, and when the resulting game is the same as what is analyzed in our article. The high-type defendant would have an incentive to provide such a hard public evidence of her type as it will change the common prior belief of D being a high type (denoted by ρ) in her favor, which in turn raises her expected payoff under arbitration, thus her settlement payoff as well. However, all the results of our analysis will remain qualitatively unchanged as long as this additional public signal is imperfect.³⁶

APPENDIX

A.1 *Proof of Lemma 1.* Proof: First note that $t^E(l) < t_A(l) < t_A(h) < t^E(h)$. This implies that for all $t \in (t_A(l), t_A(h))$ we have $u'(t, l) + v'(t) < 0$ and $u'(t, l) > 0$. Therefore, because both u and $u + v$ are concave functions, the certainty-equivalent action for the joint welfare is lower than the certainty equivalent action for D_l 's welfare. This implies that $t_l^{\max} > t_l^{\min}$.

To prove that $t_h^{\max} > t_h^{\min}$, we cannot use the above argument for proving $t_l^{\max} > t_l^{\min}$ as $u'(t, h) + v'(t) > 0$ for $t < t^E(h)$. We can still prove this as follows: Let a be the conditional probability that the arbitrator assigns to $t^A(h)$ given that the true type is h . D_h prefers settlement to arbitration iff the proposed t satisfies

$$u(t; h) \geq au(t^A(h); h) + (1 - a)u(t^A(l); h),$$

Moreover, given $\theta = h$, C prefers settlement iff

$$v(t) \geq av(t^A(h)) + (1 - a)v(t^A(l)).$$

Letting $a_D(t)$ and $a_C(t)$ denote the levels of a that satisfy the above two inequalities with equalities, respectively, we will have

$$a_D(t) \equiv \frac{u(t; h) - u(t^A(l); h)}{u(t^A(h); h) - u(t^A(l); h)},$$

$$a_C(t) \equiv \frac{v(t^A(l)) - v(t)}{v(t^A(l)) - v(t^A(h))}.$$

³⁴ In our simple bargaining setup, C would not have any incentive to acquire a costly signal because all the rents from settlement accrue to D who makes a take-it-or-leave-it offer of settlement. Under other bargaining setups, such as alternating-offers bargaining, in which the parties may share the gains from settlement, C would have an incentive to acquire costly private signals, but continue to have no incentive to acquire costly public signals. Regardless of how the signals were acquired, a public signal would lose its informational value in the equilibrium as we have shown in this article.

³⁵ We appreciate the comment of a referee to address this issue.

³⁶ If the hard evidence can prove the defendant's type, then settlement bargaining becomes a game with complete information.

Note that $a_D(t)$ and $a_C(t)$ strictly increase from 0 to 1 as t increases from $t^A(l)$ to $t^A(h)$. Also note that t that satisfies $a_D(t) = a$ is equal to t_h^{\min} . Similarly, t that satisfies $a_C(t) = a$ is equal to t_h^{\max} . Given these equalities, if $a_D(t) - a_C(t) > 0$ for $t \in [t^A(l), t^A(h)]$, then $t_h^{\max} > t_h^{\min}$ for all $a \in (0, 1)$.

Thus, it remains to prove $a_D(t) - a_C(t) > 0$ for $t \in (t^A(l), t^A(h))$. Recall that both $u(t; h)$ and $u(t; h) + v(t)$ are increasing and concave functions in this range of t whereas $v(t)$ is a decreasing one in t . If $a'_D(t) - a'_C(t) > 0$ at $t = t^A(l)$, $a'_D(t) - a'_C(t) = 0$ only once at some $t \in (t^A(l), t^A(h))$, and $a'_D(t) - a'_C(t) < 0$ at $t = t^A(h)$, then we should have $a_D(t) - a_C(t) > 0$ for $t \in (t^A(l), t^A(h))$ as $a_D(t^A(l)) = a_C(t^A(l)) = 0$ and $a_D(t^A(h)) = a_C(t^A(h)) = 1$.

Now, we prove that $a'_D(t) - a'_C(t) > 0$ at $t = t^A(l)$, $a'_D(t) - a'_C(t) = 0$ only once at some $t \in (t^A(l), t^A(h))$, and $a'_D(t) - a'_C(t) < 0$ at $t = t^A(h)$ as follows: If one draws the graphs of $\frac{\partial u(t; \theta=h)}{\partial t}$ and $-\frac{\partial v(t)}{\partial t}$ on $t \in [t^A(l), t^A(h)]$, the former graph is located above the latter one, with the value of the slope of the former one being smaller than that of the latter one (this last inequality results from the concavity of $u(t; \theta = h) + v(t)$). The graphs of $a'_D(t)$ and $a'_C(t)$ are obtained by multiplying a certain positive constant value to $\frac{\partial u(t; \theta=h)}{\partial t}$ and $-\frac{\partial v(t)}{\partial t}$, respectively, so that the resulting areas below $a'_D(t)$ and $a'_C(t)$ over $t \in [t^A(l), t^A(h)]$ are both equal to one. Because $u(t; \theta = h)$ increases more than $v(t)$ decreases over $[t^A(l), t^A(h)]$, such a constant value to be multiplied has a smaller value for $\frac{\partial u(t; \theta=h)}{\partial t}$ than the one for $-\frac{\partial v(t)}{\partial t}$. This implies that the value of the slope of $a'_D(t)$ is smaller than that of $a'_C(t)$, which in turn implies that $a'_D(t)$ and $a'_C(t)$ cross only once over $[t^A(l), t^A(h)]$, if they ever cross at all. $a'_D(t) - a'_C(t) > 0$ at $t = t^A(l)$ because $a'_D(t) - a'_C(t) \leq 0$ contradicts with both areas below $a'_D(t)$ and $a'_C(t)$ being equal to one. Given $a'_D(t) - a'_C(t) > 0$ at $t = t^A(l)$, $a'_D(t) - a'_C(t) \geq 0$ at $t = t^A(h)$ also lead to a contradiction with both areas below $a'_D(t)$ and $a'_C(t)$ being equal to one.

A.2 Proof of Proposition 1. We first show that:

LEMMA A.1. *Under the settlement bargaining game with a public signal, pooling Perfect Bayesian Equilibriums (PBEs) fail to be Universally Divine.*

PROOF. Following closely the line of reasoning in Reinganum and Wilde (1986, p. 566), suppose there is a pure pooling equilibrium at t_p . Note that $t_p < t_h^{\max}$, as arbitration will be invoked for sure, otherwise. Under such an equilibrium both defendant types propose t_p , which is then accepted by the complainant. Now consider a deviation to some settlement proposal t , where $t \in (t_p, t_h^{\max}]$. Define $b(\theta, t)$ as the probability of invoking arbitration that keeps a defendant of type θ indifferent between t and the equilibrium settlement t_p , namely,

$$u(t_p, \theta) = b(\theta, t)u_A(\theta) + [1 - b(\theta, t)]u(t, \theta)$$

or

$$b(\theta, t) = \frac{u(t_p, \theta) - u(t, \theta)}{u_A(\theta) - u(t, \theta)}.$$

Notice that $b(h, t) > b(l, t)$ from the assumptions of $u(t, l) - u_A(l) > u(t, h) - u_A(h)$ and $\frac{\partial u}{\partial t} > 0$, which implies that the minimum rejection probability that prevents D_h from deviation to t is greater than that for D_l .³⁷ In other words, in the language of Banks and Sobel (1987), D_h is the type most likely to deviate to t . Universal Divinity requires the uninformed

³⁷ In fact, the assumption 4 (a weaker condition) is sufficient to yield this result. With regard to pooling PBEs under which arbitration rises with a positive probability as well as mixed-strategy pooling PBEs, we can show that such PBEs also do not survive the Universal Divinity refinement given the assumption 4. Beshkar and Park (2019) provide a proof for such a result under a private signal, which one can easily adjust to prove a corresponding result under a public signal.

party to believe that an out-of-equilibrium proposal comes from the type most likely to deviate. Therefore, if t is proposed, C will believe that D is of h type. Therefore, if $t \in (t_p, t_h^{\max}]$, will accept the proposal with certainty. A similar argument show that partial pooling equilibria are not Universally Divine either. \square

Now we show that only the fully-separating equilibrium having $\beta(t)$ satisfy (8) with equality is Universally Divine, under which the proposal of $D_\theta, t(\theta)$, must be equal to t_θ^{\max} .³⁸ Note that such a fully-separating equilibrium is a separating equilibrium that maximizes the expected payoff of a high-type defendant given the assumption in (4). Utilizing this fact together with the expected payoff of a low-type defendant being $u(t_l^{\max}; l)$ under any separating equilibrium, we can show that any partially-separating equilibrium with $t(h) \in [t_h^{\min}, t_h^{\max})$ does not survive the Universal Divinity refinement: There exists an off-the-equilibrium proposal, $t' \in (t(h), t_h^{\max}]$ so that $b(h, t') > b(l, t')$, which in turn makes such deviation profitable for a high-type defendant as shown above in Lemma A.1.

To prove that the fully-separating equilibrium with $t(h) = t_h^{\max}$ does survive the universal divinity refinement, now consider an off-equilibrium proposal $t \in [t_h^{\min}, t_h^{\max})$ and let $b(\theta, t)$ be the probability of invoking arbitration that keeps a defendant of type θ indifferent between t and t_θ^{\max} . Given the assumption in (4), it is straightforward to show that $b(h, t) < b(l, t)$, which implies that the off-equilibrium proposal, t , has been offered most likely by D_l . As a result C will reject such a proposal with certainty. With similar logic, we can show that a separating PBE that makes D_l strictly better off by separating from D_h is not Universally Divine. Therefore, C 's equilibrium rate of arbitration when $t(h) = t_h^{\max}$ is proposed must make D_l indifferent between proposing t_l^{\max} and t_h^{\max} , that is, $\beta(t_h^{\max})$ must satisfy (8) with equality.

Finally, C will accept t_l^{\max} for sure under a separating PBE. Otherwise, D_l may profitably deviate by proposing $t (< t_l^{\max})$ that is close enough to t_l^{\max} .

A.3 Proof of Proposition 2. The result that only the fully-separating equilibrium having $\beta(t)$ satisfy (8) with equality is Universally Divine (Lemma A.1 and Proposition 1) can be extended to the case of private signal.³⁹ Part (i) of Proposition 2 follows this observation immediately because a fully-separating equilibrium requires the proposal of D_θ to be $t(\theta) = t_\theta^{\max}$.

To prove part (ii) of this proposition note that under a separating equilibrium, D 's true type is revealed and, therefore, the only way to make C indifferent between settlement and arbitration when $\theta = h$ is to propose t_h^{\max} . Moreover, the condition in (10) must be satisfied with equality to make D_l indifferent between proposing t_l^{\max} and t_h^{\max} , a condition that needs to hold to satisfy the Universal Divinity that requires the equilibrium to maximize the expected payoff of the high-type defendant. Finally, C will accept t_l^{\max} for sure under a separating PBE because otherwise, D_l may profitably deviate by proposing $t (< t_l^{\max})$ that is close enough to t_l^{\max} .

To prove part (iii) of Proposition 2, first note that

LEMMA A.2. $\beta_h(1 - \beta_l) = 0$.

PROOF. If $\beta_h(t) \in (0, 1)$ in equilibrium, then C_h must be indifferent between settling at t and arbitration, namely:

$$v(t) = \Pr(\theta = h | \theta^C = h) v_A(h) + \Pr(\theta = l | \theta^C = h) v_A(l),$$

³⁸ C must be indifferent between accepting the proposal of the high type, $t(h)$, and arbitration in a separating equilibrium, under which the low-type defendant's incentive to propose $t(h)$ is deterred by a positive probability of arbitration. Therefore, we must have $t(h) = t_h^{\max}$ under a full-separating equilibrium because it is the only settlement proposal that makes C indifferent between settlement and arbitration knowing that $\theta = h$. Also note that $t(l) = t_l^{\max}$ makes C indifferent about settlement knowing that $\theta = l$.

³⁹ Beshkar and Park (2019) provide a formal proof for this result.

where $\Pr(\theta = h|\theta^C = h) = 1 - \Pr(\theta = l|\theta^C = h) = \frac{\gamma\rho}{\alpha(1-\gamma)(1-\rho)+\gamma\rho}$. Solving for α yields

$$(A.1) \quad \alpha = \frac{\gamma\rho}{(1-\gamma)(1-\rho)} \frac{v(t) - v_A(h)}{v_A(l) - v(t)}.$$

Similarly, if $\beta_l(t) \in (0, 1)$ in equilibrium, the incentive compatibility of C_l requires

$$(A.2) \quad \alpha = \frac{(1-\gamma)\rho}{\gamma(1-\rho)} \frac{v(t) - v_A(h)}{v_A(l) - v(t)}.$$

Conditions in (A.1) and (A.2) cannot be satisfied simultaneously. In particular, whenever C_h is indifferent between settlement and arbitration, C_l will strictly prefer arbitration. Conversely, if C_l is indifferent, C_h strictly prefers to settle. \square

If α satisfies the condition in (A.1), it would be increasing in γ and it tends to infinity for $\gamma \rightarrow 1$. Conversely, if α satisfies the condition in (A.2), it will be decreasing in γ . Therefore, there is $\bar{\gamma}$ below (above) which only the condition in (A.1) (respectively, (A.2)) can be satisfied with $\alpha \leq 1$. Finally, note that the probability of arbitration against an imposter is equal to $1 - \gamma\beta_l(t_h^{\max}) - (1-\gamma)\beta_h(t_h^{\max})$, which must make D_l indifferent between proposing t_l^{\max} and t_h^{\max} . This equilibrium probability must remain constant as γ changes. Therefore, given that $\beta_l(t_h^{\max}) > \beta_h(t_h^{\max})$, we must have $\partial\beta_h(t_h^{\max})/\partial\gamma \leq 0$ and $\partial\beta_l(t_h^{\max})/\partial\gamma \leq 0$, with at least one inequality being satisfied strictly. This completes the proof.

A.4 PROOF OF PROPOSITION 3. First note that regardless of his information, C 's settlement proposal, t , must be either t_l^{\min} or t_h^{\min} because the response of either type of D will be identical for any offer between (t_l^{\min}, t_h^{\min}) , and C will always prefer these extreme offers to any point in the middle.

If $t = t_l^{\min}$, then D_l will settle and D_h will litigate. Therefore, given θ^C , C 's expected payoff from proposing $t = t_l^{\min}$ is

$$\Pr(\theta = h|\theta^C) v_A(h) + [1 - \Pr(\theta = h|\theta^C)] v(t_l^{\min}).$$

If $t = t_h^{\min}$, then both types of D will accept the proposal in which case the payoff of C is given by $v(t_h^{\min})$. Therefore, C will propose $t = t_h^{\min}$ if and only if

$$v(t_h^{\min}) \geq \Pr(\theta = h|\theta^C) v_A(h) + [1 - \Pr(\theta = h|\theta^C)] v(t_l^{\min}),$$

or, equivalently, iff

$$\Pr(\theta = h|\theta^C) \geq \frac{v(t_l^{\min}) - v(t_h^{\min})}{v(t_l^{\min}) - v_A(h)}.$$

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